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A Surrogate Time Series Model for the Kp Geomagnetic Index

August 20, 2013

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Space Science Applications Laboratory
Physical Sciences Laboratories

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Approved by:

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Abstract

A new surrogate time series model for the geomagnetic index, Kp, is presented. This model can be used to generate arbitrarily long sequences of the 3-hour Kp index with the observed long-term distribution of Kp as well as observed temporal correlations to within 0.1. The primary application for this model is the simulation of future scenarios of Kp as inputs to other geophysical models, such as a magnetic field model, for satellite mission planning purposes.

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1. Introduction

The geomagnetic index Kp is widely used for parameterizing geophysical processes. It is a 3-hour index derived from the range of variation in the horizontal (northward) component of the magnetic field at 12 low-latitude ground-based magnetometer stations. The index ranges from 0 to 9 with + and – designators used to indicate 1/3 integer resolution.

Kp has been used to characterize all manner of geophysical phenomena in climatological and physical models. Therefore, a surrogate time series of Kp is useful for generating realistic scenarios of the outputs of those physical models. For the purposes of this report, we will focus on the example of using Kp as an input to the Tsyganenko 1989 (T89) magnetic field model.* T89 represents the magnetic field associated with electrical currents flowing in space (external to the Earth). It requires time, location, and Kp as inputs. Kp controls the intensity of those external currents in the model. While T89 has been superseded by more sophisticated parametric models, it is still widely used because it requires only the Kp parameter as input.

We emphasize that the model we are about to describe is essentially a Monte Carlo model of the time evolution of Kp. It does not include any physics, and it does not represent any real sequence of geomagnetic activity. Rather, it reconstitutes realistic variations in the Kp magnetic index for use in driving other parametric geophysical models.

* Tsyganenko, N. A., "A magnetospheric magnetic field model with a warped tail current sheet," *Planet. Space Sci*., **37**(5), 1989.

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2. Formulation of the Model

We formulate the model as a table of transitions probabilities between the 28 discrete states of the Kp index. The transition probability is governed by the Kp values at previous times: 1, 2, and 8 periods ago, or, equivalently, 3, 6, and 24 hours ago. At lags of 3 and 6 hours, the Kp index is collapsed onto the integers 0−8 using a floor operation (e.g., 0, 0^+, and 1^- all become 0), with Kp values of 9 also being grouped with 8−9^- as 8. At 24-hours lag, the index is collapsed onto 3 discrete intervals: 0−3^-, 2−6^-, and 6−9. The model, therefore, consists of a table of probability for the current Kp given appropriately collapsed prior Kp values. In the typical notation of conditional probability, the model is given as a table of P(Kp(t)|Kp(t-1),Kp(t-2),Kp(t-8)), or, Kp at time t conditioned on Kp at 1, 2, and 8 prior 3-hour time steps.

The probabilities in the table are derived from Kp observed over the interval 1963 to 2006.

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3. Demonstration

To "run" the model, assumes Kp has been 2 (its median) for the past 8 time periods.

To generate the next Kp, one uses the look-up table to obtain the probabilities for the 28 possible values of the next Kp given the recent history of Kp. One then generates a random number *u* uniformly drawn from the real numbers 0 to 1. One then computes a running sum of the 28 probabilities starting at Kp = 0 until the running sum matches or exceeds *u*, and accepts the associated value of Kp as the next Kp. The random numbers *u* must be uncorrelated from one time to the next, as is typically the case with random number generators.

One runs the model in this fashion for 29 days to get its time history adequately "warmed up." After the warm-up period, the model can be run for arbitrarily long times to obtain a surrogate time series of Kp. Figure 1 shows ten realizations of the model, with three highlighted as examples.

Note: it is also possible to us an observed sequence of Kp for the first 8 time periods, in which case no warm up is required, and the model will produce scenarios that could realistically follow the observed sequence of 8 Kps. This approach could be used for contingency planning in real-time applications.

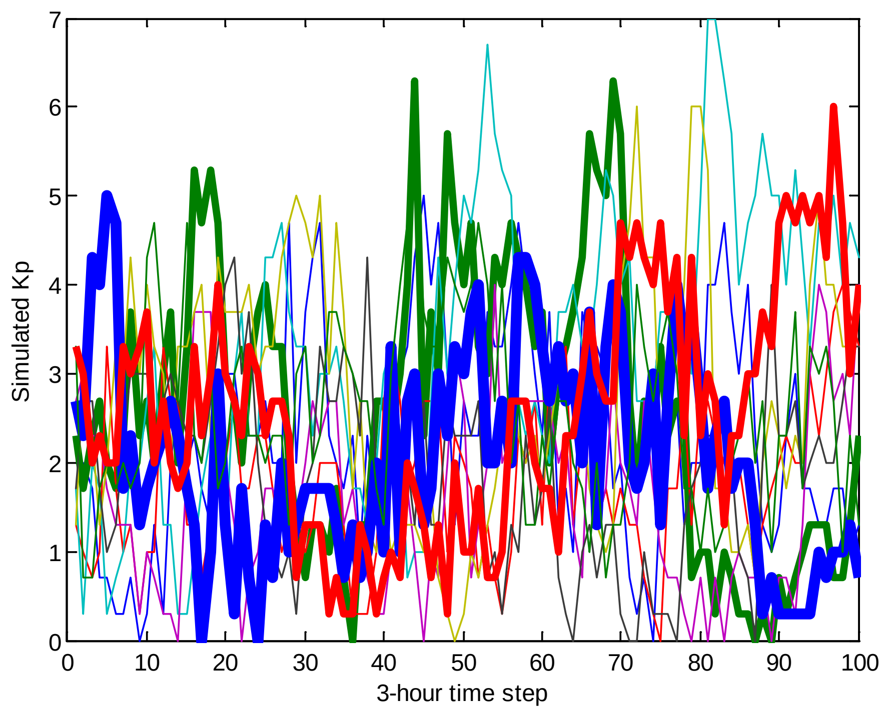

Figure 1. Ten realizations of simulated Kp. Three arbitrarily selected realizations are emphasized.

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4. Validation

To validate the model, we generated 10 scenarios for 1000 time steps and computed (1) the cumulative distribution of Kp, and (2) the lag correlation. Figure 2 shows that the simulated cumulative distribution of Kp very closely matches the observed distribution, to within a few percent at any Kp value. Figure 3 shows that the model reproduces the correlation coefficient to within 0.1 at all lags. However, whereas the simulated Kp lag correlation drops to 0 at approximately 35 time steps(~4 days), the observed Kp correlation drops off to 0.1 rather than 0 at long lags (80 time steps is 10 days). While it is technically possible to add still more time lags to the model to achieve still better agreement with the observed lag correlations, we expect that the current formulation is “good enough” for most applications where long-term, weak temporal correlations are probably of little significance.

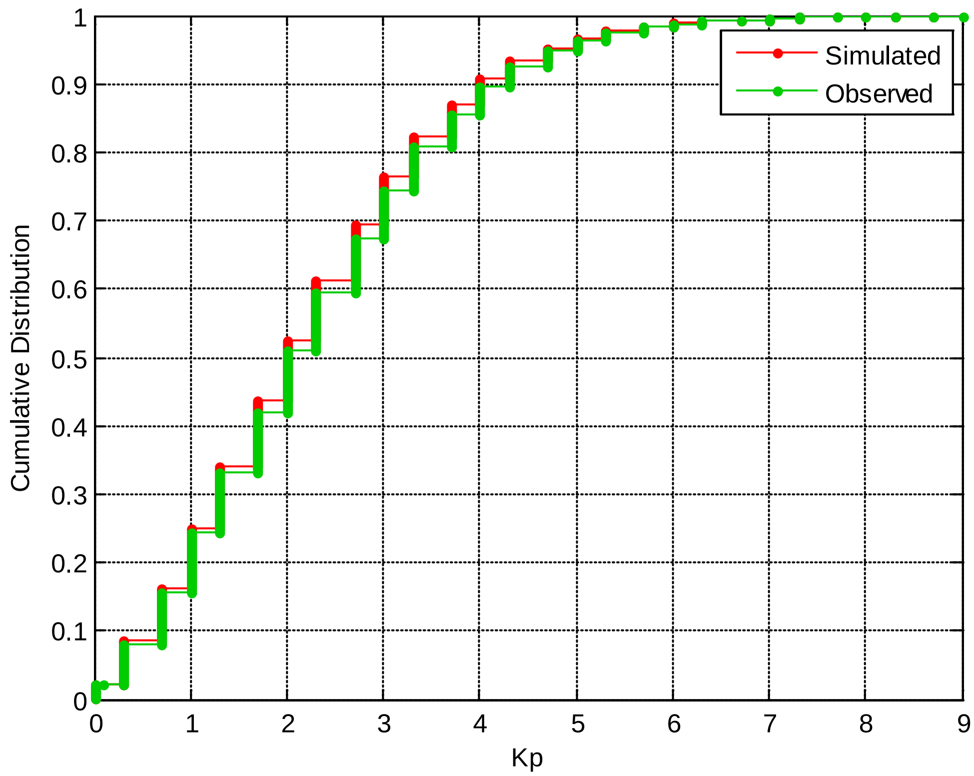

Figure 2. Comparison of observed and simulated cumulative distributions of Kp.

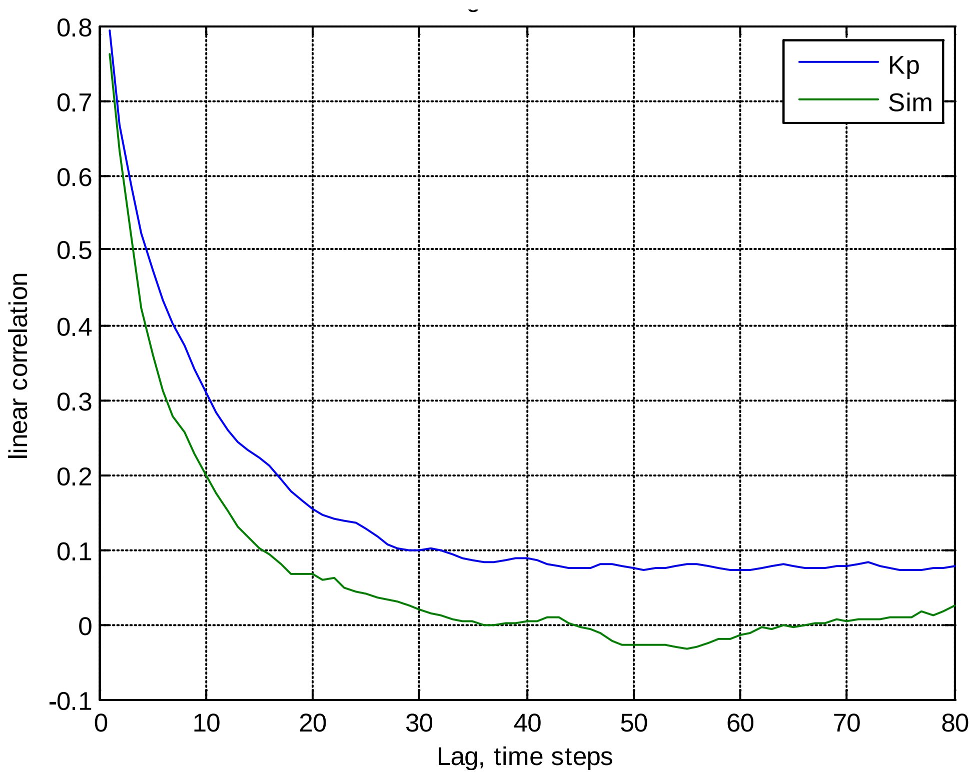

Figure 3. Comparison of lag correlations for observed Kp and the simulation.

5. Summary

We have developed a surrogate time series model of the geomagnetic index Kp. The model can be used to generate an arbitrary number of arbitrarily long time series of Kp. These simulated series will have the same statistical distribution as Kp to within a few percent and will have approximately the same temporal correlations, to within about 0.1.

The surrogate time series of Kp can be used either to generate arbitrary scenarios of geophysical models that depend on Kp, or the model can be seeded with observed Kp to generate realistic near-term contingency scenarios.