2.3 AE9 Templates Derived from AE8

This section describes the calculations used to create templates for AE9 V1.0/V1.1 derived from the AE8-MIN and AE8-MAX models. Templates are used to fill in energy and spatial gaps in the data sources used to create AE9. AE8 is a valuable source of information regarding what is a reasonable shape for the radiation belts, and therefore was a significant source of the templates used in AE9.

In the AE9/AP9 models, templates are shapes used during construction of the runtime tables to fill in gaps the parameter maps. Each template is an estimate of the $\log_{10}$ flux as a function (or table) of the model coordinates. In the case of AE9, the AE8 model itself was used to generate some of these templates. This document describes how AE8 data was manipulated to provide templates for AE9.

This section has also been released as Aerospace Report No. TOR-2014-00292.

2.3.1 K-h_{min} templates

We constructed a set of AE9 templates from AE8 in the low altitude, K-h_{min}, coordinate system. We did this for both AE8-MIN and AE8-MAX [*Vette*, 1991b].

First, we sampled the AE8 model, obtaining the differential, omnidirectional flux at the AE9 energy grid on a spatial grid: altitudes from 0 to 1000 km in steps of 50 km, longitudes from -180° to 180° in steps of 10°, and latitude from -80° to 80° in steps of 1°. We performed this sampling at the reference date of midnight UTC on 1 June 1968, which is roughly the midpoint of the data sets that were used to construct AE8.

For each point in the alt/lat/lon grid, we also computed the K, h_{min} coordinates for the AE9 model using the Olson-Pfitzer Quiet (OPQ) magnetic field model [*Olson and Pfitzer*, 1977] added to the International Geophysicsl Reference Field model [*Finlay et al.*, 2010]. Next, we then binned the results in the K-h_{min} grid of the AE9 model, and took the median AE8 flux within each bin. We note that *we did not attempt to convert from omnidirectional to unidirectional flux* because for the low altitude grid, the locally mirroring flux dominates the omnidirectional average, and so the differences in the omni to uni correction is expected to be only weakly varying with K, h_{min}, or energy; the template needs only the relative, not the absolute, shape of the flux map.

After binning, there were still some regions of the AE9 grid that did not contain any points from AE8. We used several extrapolation strategies. First, we interpolate in energy within each K-h_{min} bin, and extrapolate to higher energy, using $\log_{10}$ flux ~E^{-n}, for n = 3, 5, 8. Next we extrapolated to smaller h_{min} using $\log_{10}$ flux ~ $\log_{10}$ (2./exp(-(h-1500)/500)+exp(-(h-600)/h_0))), for h_0 = 50, 75, 100. Finally, we extrapolated to larger K using $\log_{10}$ flux ~ $-K/K_0$, for K_0 = 2.8, 3, 3.2.

Next, we applied a smoothing mask, that smoothes over 3 adjacent K bins and 7 adjacent h_{min} bins. Then we remove points that are in the loss cone, as defined by the AE9 model grid.

Finally, we save to a file with a name such as AE9V10_ae8max_Khmin_PLnn_extrapj.mat, where $n is the power law index n, and $j specifies the h_{min}/K extrapolation option 1-9. For example, $j=2 specifies $h_0 = 50$, $K_0 = 3$ while $j=8 specifies $h_0 = 100$, $K_0 = 3$.

This completes development of the AE9 K-h_{min} templates from AE8. We note that the use of omnidirectional rather than unidirectional fluxes in building the template likely slightly smears out the spatial gradients – which we do explicitly as well when we apply the smoothing. Figure 18 provides a sample K-h_{min} template for AE9 derived from AE8, at 750 keV.

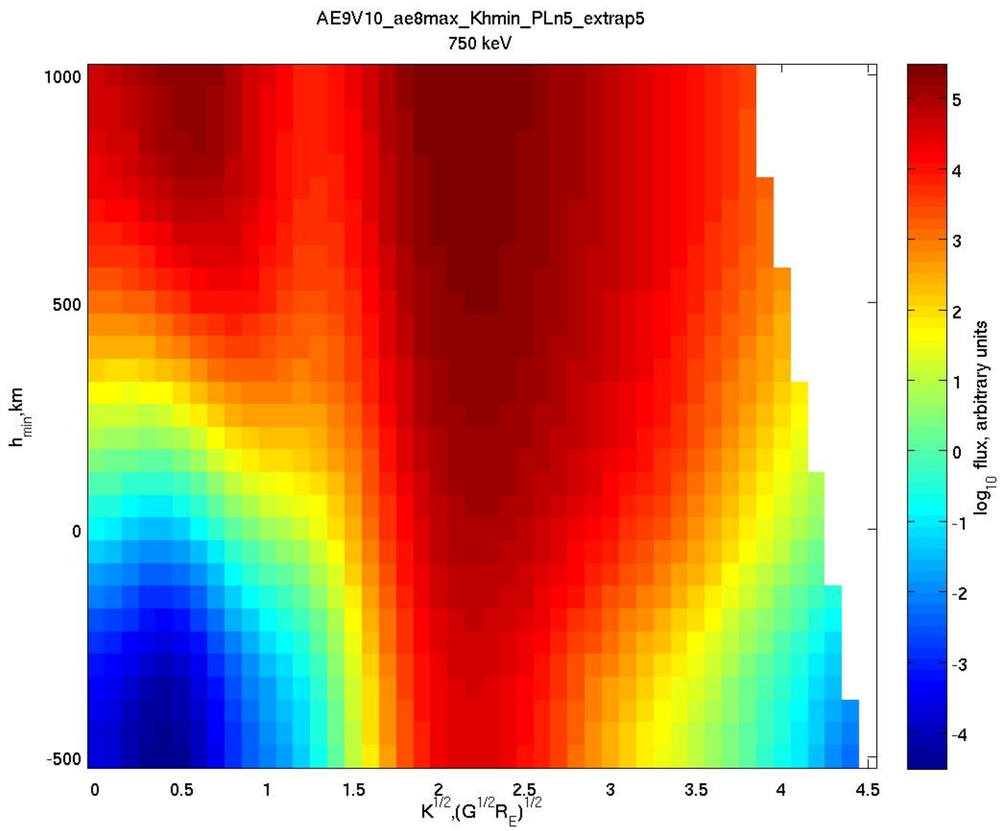

Figure 18. An AE9 V1.0 *K-*h_{min} template at 750 keV derived from AE8-MAX. The deep red region at top left is the inner zone, and the deep vertical red region in the middle is the outer zone. The atmospheric loss cone is at h_{min} = 100 km, so this template implies significant quasi-trapped flux (h_{min} < 0).

2.3.2 *K*-Φ templates

As with the K-h_{min} grid, we developed templates from both AE8-MIN and AE8-MAX for the high altitude K-Φ grid for AE9. However, because high altitude flux is not nearly as dominated by the locally mirroring flux, it was necessary to convert from AE8's omnidirectional fluxes to unidirectional fluxes to make consistent template.

We begin by looping over the energies and L shells of the AE8 model (AE8-MIN or AE8-MAX) being used to build the AE9 template. For each energy and L, AE8 provides a set of omnidirectional fluxes as a function of B/B_0, which we convert to unidirectional fluxes as a function of equatorial pitch angle, as described in Appendix G. (B/B_0 is the ratio of the local magnetic field strength to the minimum field strength on the same field line). We interpolate the resulting equatorial pitch angle distribution over any local minima with respect to angle.

We then convert from *E*, equatorial pitch angle, and *L* into AE9 model coordinates. To convert from *L* to Φ, we use $\Phi = 2\pi 0.31153/L$, which is appropriate for epoch 1 Jan 2000. We convert from equatorial pitch angle and *L* to *K* using dipole relations given in *Schulz and Lanzerotti* [1974; pages 51-52].

Next we interpolate the AE8 sample onto the AE9 grid. For each energy channel, we do a 2-D interpolation, where the *K* dimension is transformed to $K^{1/2}$ and scaled onto the range [0,1], and the Φ dimension is transformed to $\log_{10}\Phi$ and also scaled onto [0,1]. The interpolation is done in a log flux sense, so any 0 fluxes are replaced by half the smallest positive flux before the log flux is taken.

Next, we numerically differentiate the flux spectrum to obtain unidirectional, differential fluxes versus energy, equatorial pitch angle, and *L*. We interpolate onto the AE9 energy channels and then interpolate over any local minima in the spectrum. Next we filter out any fluxes that are 10 orders of magnitude smaller than the largest flux.

Next we interpolate in *K* at fixed energy and Φ to fill any gaps. We do not interpolate in Φ because that could corrupt the 2-belt structure.

Then we use one of 3 power law functions to extrapolate to higher energies: flux ~ E^{-n} for $n = 3$, 5, and 8. Then we extrapolate to lower Φ using one of 3 functions: $\log_{10}$ flux ~ $-\Phi_0/\Phi$, for $\Phi_0 =$ 1.5, 1.7, 1.9. Finally, we extrapolate to higher *K* using one of 3 functions: $\log_{10}$ flux ~ $-K/K_0$, for $K_0 = 2.5, 3, 3.5$.

At this point, we make an ad hoc extrapolation to the highest few Φ and *K* in the inner zone. For each energy, we start at the lowest (actually, 2nd to last) Φ grid point for which there is a gap and, for each *K*, we copy the next lower Φ grid point at one higher *K*. Using this copy-and-offset approach at fixed energy, we work our way up to the upper limit in Φ. This ad hoc algorithm was selected because it seemed, visually, to extend the pattern in the template.

Then, we smooth with a mask that extends 15 grid points in *K* and 5 grid points in Φ. We remove any points in the grid's loss cone.

Finally, we save the template to a file named AE9V10_ae8min_KPhi_PLnn_extrapj.mat, where, as before, $n is the power law exponent, and $j is 1 to 9. For $j=2, $\Phi_0 = 1.7$ and $K_0 = 2.5$; for $j=8, $\Phi_0 = 1.9$ and $K_0 = 2.5$

Figure 19 shows an example of a *K*-Φ template for AE9 V1.0 derived from AE8.

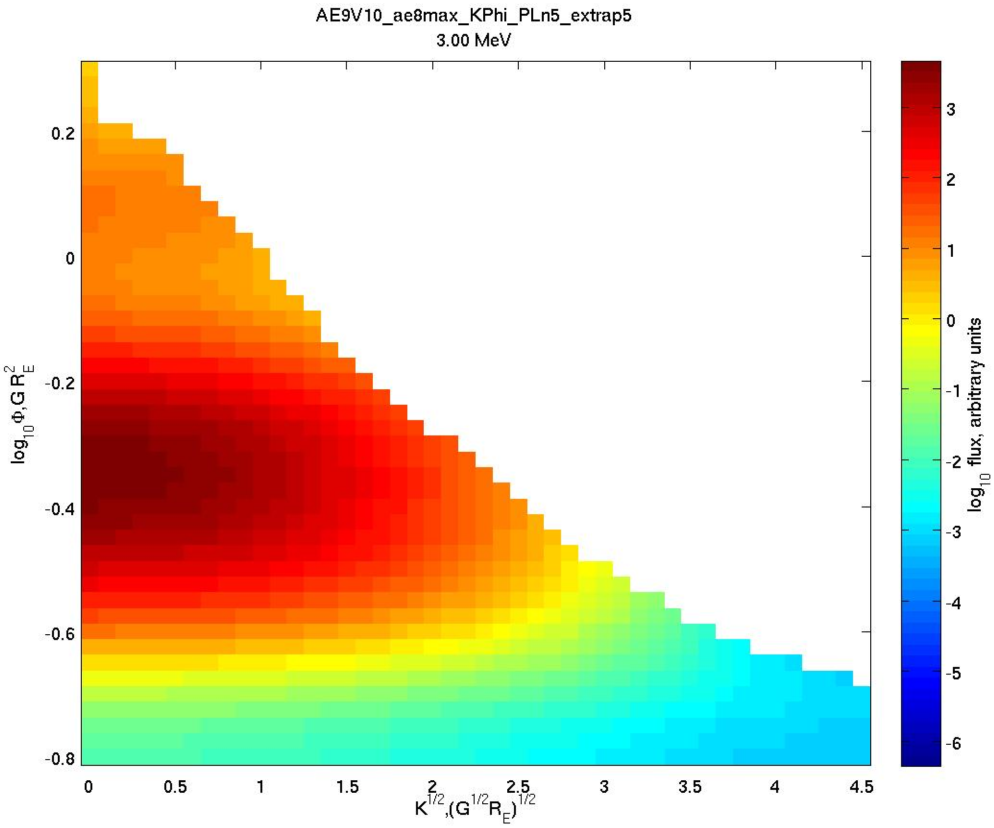

Figure 19. An AE9 V1.0 *K*-Φ template at 3 MeV derived from AE8-MAX. The orange region at top left is the inner zone, and the dark red region is the outer zone.

2.3.3 Summary

We have described the process by which the AE8 model is used to develop templates for filling in the spatial gaps in the AE9 model. This development requires several best guess and ad hoc assumptions – which is the very purpose of having templates, to incorporate that prior knowledge and uncertainty about the radiation belt topology into AE9.

There are several known issues with the conversion of AE8 into AE9 templates. First, there is an inconsistency in the epoch between the AE8 data, taken mainly in the 1960s and 1970s and the intended use of AE9 in the 2010s and beyond. This mostly affects the low altitude grid, since h_{min} is not a true adiabatic invariant, but merely a drift invariant.

Also, as mentioned above, the K-h_{min} template is built using omnidirectional rather than unidirectional fluxes. This is expected to somewhat smooth the template. We include explicit smoothing as well, so the omni- versus unidirectional flux issues is likely very minor.

In the construction of the K-Φ template, some dipole relationships are used. These are not quite correct for the OPQ field model. Further the conversion from omnidirectional to unidirectional flux uncovered some non-physical angular distributions implied by the AE8 maps. Similarly, the differentiation from integral to differential flux revealed some non-physical spectral shapes. In both cases we interpolated over unphysical local minima.

Because the AE8 templates are used only as one of the ways that AE9 fills in the spatial gaps, the issues with the templates should manifest, if at all, only as part of the error bars in the final AE9 maps, and then only in regions where there is inadequate data.