2.2 AP9 Templates

Templates for AP9 were developed by combining data from several spacecraft and sensors. The construction of the templates makes use of the empirical observation that proton flux spectra over a large spatial range are described quite well by a sum of exponential spectra of the form

$$j = A_{low} \exp\left(-\frac{E}{E_{low}}\right) + A_{high} \exp\left(-\frac{E}{E_{high}}\right)$$
(77)

where E_{low} is of the order of 0.1 MeV and E_{high} is of the order of 50 MeV. The construction of the templates relied on fitting spectra to the available flux data at a given spatial bin; the four fitting parameters were A_{low} , A_{high} , E_{low} , and E_{high} . This observation simplified the process of constructing templates in two ways. First, it provided a rational means for extrapolating observed spectra in energy. Second, when interpolating or extrapolating in spatial coordinates, it was only necessary to do so for four parameters rather than 20 or so energies. Slightly different procedures were used for the K-h_{min} and the K- Φ templates; these are described below.

2.2.1 K-h_{min} templates

The *K*- h_{min} templates are based on data from three sensors: S3-3/Telescope, CRRES/PROTEL, and TSX-5/CEASE. These three sensors cover nearly the entire AP9 energy range, but not generally at all spatial locations within the *K*- h_{min} grid. The data are combined as follows:

- 1) Data from each sensor are binned in the appropriate grid. A separate file is created for each bin and for each sensor.
- 2) For each sensor, and each bin file, the 25th, 50th, 75th, and 95th percentile fluxes at each energy are determined. These values are written back into the bin files.
- 3) For each sensor, the bin files are scanned and combined to form a "3Dmap," which is a MATLAB structure with a format similar to the template structure, but only contains data from one sensor. At this point, the fluxes are interpolated from the "native" energies of the sensor to energies corresponding to the AP9 energy grid. The 3Dmap is essentially a map of the flux measured by a given sensor.
- 4) 3Dmaps from separate sensors are combined into a preliminary template structure. Spectral fits are obtained for each bin; the four fitting parameters are determined using a least-squares fitting routine. In many cases there are not enough points for a true least-squares fit. In these cases a nominal value of E_{low} or E_{high} is assumed, and A_{low} or A_{high} is calculated directly from the available data points. The four fitting parameters for each bin are saved in the template structure. Figure 11 shows an example of the data and spectral fit in one bin.
- 5) The four fitting parameters are smoothed, interpolated, and extrapolated using a twodimensional smoothing algorithm that allows boundary conditions to be specified. The boundary conditions are determined by visual inspection of the un-smoothed parameters

and represent best estimates of how the parameters vary. Figure 12 and Figure 13 show examples of E_{tow} before and after smoothing.

- 6) The smoothed fitting parameters are used to compute energy spectra spanning the full energy range of 0.1 to 400 MeV in each bin. The resulting flux map is saved as a "nominal" template.
- 7) In order to account for uncertainty in the spectral shapes and to introduce variability in the templates, modified spectra are also computed by assuming power-law spectra at the low and high ends of the spectrum. Power-law extensions at the high energy end are computed using assumed exponents of 3 and 5. The power-law extension begins at the energy where the slope of the exponential spectrum equals the slope from the power-law spectrum. The power-law extension at the low-energy end assumes an exponent of 10 and is computed for energies less than 0.6 MeV.
- 8) Steps (3) through (7) above are repeated for the 25th, 50th, 75th, and 95th percentiles.

2.2.2 *K*-Φ templates

The *K*- Φ templates are based on data from three sensors: Polar/IPS, CRRES/PROTEL, and TSX-5/CEASE. These sensors cover nearly the entire AP9 energy range, but not generally at all spatial locations within the *K*- Φ grid.

The construction of the K- Φ templates is essentially the same as for the K- h_{min} templates through steps (1) through (4) above, except that the binning is done using the K- Φ grid. A somewhat different procedure is used to smooth the fitting parameters and generate variability in the templates.

- 5) The flux map at each energy is adjusted so that fluxes are decreasing in *K* and there are no "lumps' or "islands" of high flux:
 - a. Fluxes are forced to be monotonically decreasing in *K* by fitting a $j=\sin^n(\alpha)$ pitch angle distribution.
 - b. Fluxes are forced to be monotonically decreasing in Φ ; this is more complicated because there may really be local maxima (e.g., the CRRES second belt). For each value of *K*, a value Φ_{max} is defined; for $\Phi < \Phi_{\text{max}}$, the flux is forced to be monotonically decreasing. Points in the grid which violate the criterion are set to NaNs; then the values are filled in by smoothing.
- 6) Spectral fits are performed for the adjusted flux map.
- 7) The fitting parameters are smoothed and extrapolated.
- 8) In order to account for uncertainty in the spectral shapes, modified spectra are also computed by varying the fitted values of E_{low} and E_{high} ; both parameters were multiplied by factors of 2/3, 1, and 3/2, resulting in nine different combinations.
- 9) Steps (5) through (8) above are repeated for the 25th, 50th, 75th, and 95th percentiles.

2.2.3 Summary

The proton flux templates used for AP9 version 1.0 are based on spectral fits to data from several of the sensors used in populating the flux maps. Although the templates roughly reflect the expected spatial and spectral variations, they do not in any way represent models of the actual proton flux.

Figure 11 to Figure 17 show template flux maps at 1 and 50 MeV for the two coordinate systems. In addition to the figures presented in this report, a more complete set of figures showing the fitting parameters and flux maps is contained in a PowerPoint file, AP9 Template Results.ppt.

A comparison of the AP9 model with its constituent data sets indicates that a revision of the templates may be in order for newer versions of the model. In particular, the templates do not seem to reflect the transient belt observed in the CRRES/PROTEL data. It may be better to split the PROTEL data explicitly into pre- and post-storm models and develop different sets of templates for each.



Figure 11. Spectral fitting for one bin in the K- h_{min} template. Data from three sensors are as shown in the legend. The thin light blue lines are the low- and high-energy fits to the data. The thin black line with open circles is the combined fit.



Figure 12. Example of unsmoothed E_{low} as a function of K and h_{min} from curve fitting procedure.



Figure 13. Example of smoothed E_{low} as a function of K and h_{min} .



Figure 14. Final map of template fluxes at 1 MeV in the K- h_{min} coordinate system. Gray dots indicate points in the grid where actual data existed.



Figure 15. Final map of template fluxes at 1 MeV in the *K*-Φ coordinate system.



Figure 16. Final map of template fluxes at 50 MeV in the K- h_{min} coordinate system.



Figure 17. Final map of template fluxes at 50 MeV in the *K*-Φ coordinate system.