# Air Force Research Laboratory

## Spectral Inversion

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***Integrity ★ Service ★ Excellence***

# Outline

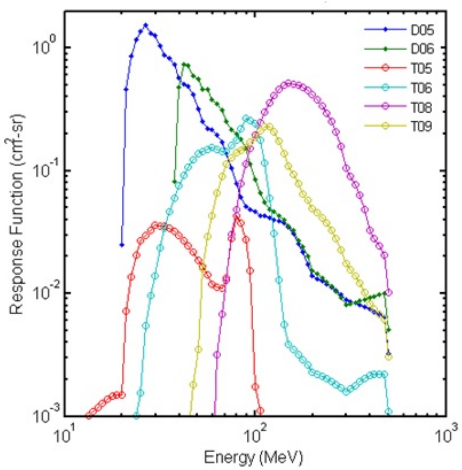
- Why spectral inversion?
- How it works
- PCA spectral model
- Angular response

# Why Spectral Inversion?

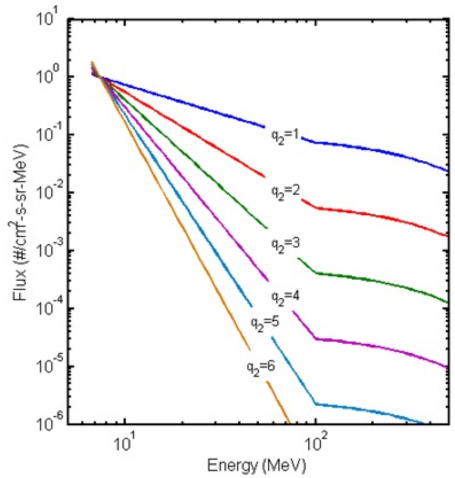
- Science-grade instruments can measure directional, differential flux with good spectral and angular resolution
- Most detectors are not science-grade
  - wide field of view
  - small numbers of integral energy channels
- Spectral inversion allows us to determine energy spectra from integral-type detectors

# Spectral Inversion:  How It Works

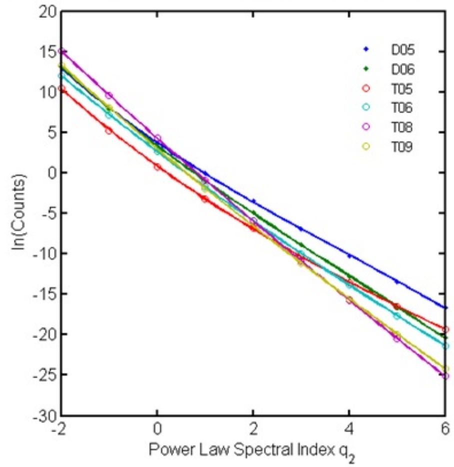
(1) Channel response functions




(2) Assume a spectral shape



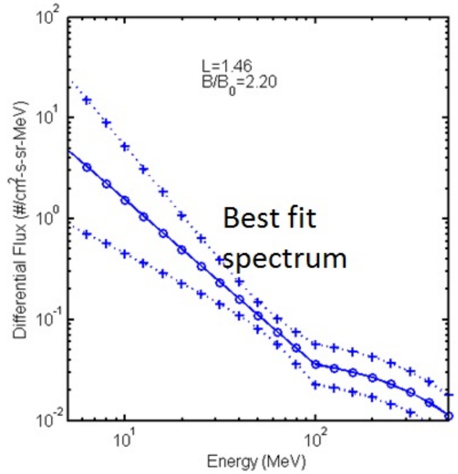

(3) Integrate (1) with (2) to obtain channel response to input spectrum

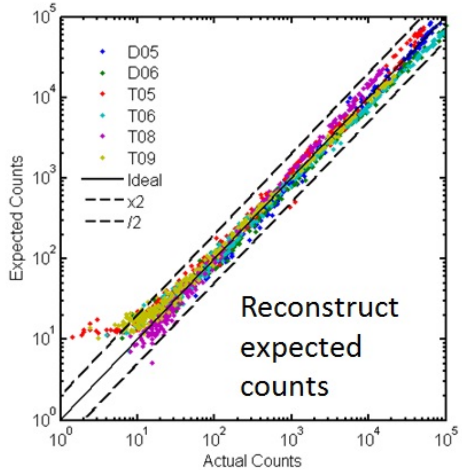



(4) Vector of Observed Counts

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix}$$

Optimization routine finds “best” q1, q2 to fit observed counts



# Problem Formulation (1)

$$C_i = \int_0^{\infty} dE \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \, A_i(E;\theta)\, j(E;\bar{\theta},\bar{\varphi})$$

$$= j_{90} \int_0^{\infty} dE \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \, A_i(E;\theta) F(E;\bar{\theta},\bar{\varphi})$$

$j_{90} =$ the locally mirroring directional, differential particle flux (e.g., in particles/cm$^2$-s-sr-MeV)

$F(E;\bar{\theta},\bar{\varphi}) =$ angle- and energy-dependent particle angular distribution function

$A_i(E;\theta) =$ the angle- and energy-dependent effective area for the i$^{th}$ channel of the detector (e.g., in cm$^2$)

$\theta, \phi =$ polar and azimuthal look directions in *detector* coordinates

$\bar{\theta}, \bar{\varphi} =$ polar and azimuthal look directions in *magnetic* coordinates

# Problem Formulation (2)

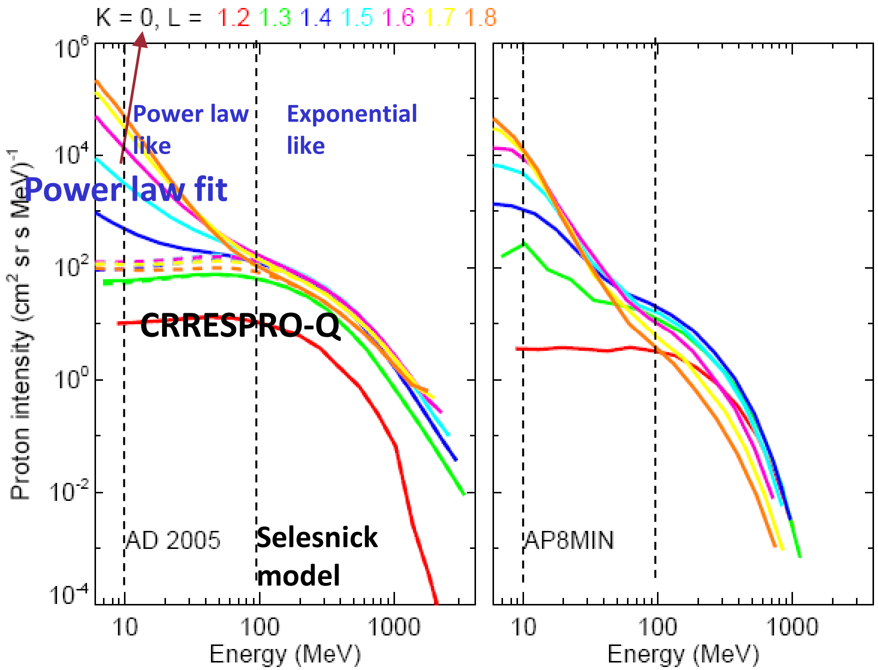
- **Recast as:**

$$\vec{y} \approx \vec{\lambda} = \delta t \int_0^\infty \vec{G}(E) f(E) dE + \vec{b}$$

$$\begin{aligned} \vec{y} &= \text{a vector of observed counts} \\ \vec{\lambda} &= \text{a vector of expected counts} \\ \delta t &= \text{integration time} \\ \vec{G} &= \text{a vector of geometric factors (response functions)} \\ f(E) &= \text{differential flux at energy } E \\ \vec{b} &= \text{a vector of background counts} \end{aligned}$$

- **Solved by parameterizing** $f(E) = f(E; \vec{q})$ **and determining maximum likelihood value of** $\vec{q}$
  - **analytical (e.g., power-law, Maxwellian, …)**
  - **discrete (e.g., PCA)**

# Proton Analytical Spectral Inversion




**From Selesnick, et al., Space Weather, 5, s04003, doi:10.1029/2006SW00275, 2007.**

Power law is a reasonable approximation between 10 – 100 MeV

$$j(E,\theta,\varphi) = b(\theta,\varphi)E^{-n},$$

Fit to exponential for *E* > MeV with fixed e-folding rate determined from Selesnick, et al. model

$$j(E) = \begin{cases} \exp(q_1 - q_2 \ln E) & ; E \le E_{break} \\ j_{break} \exp\left(-\frac{E}{E_0}\right) & ; E > E_{break} \end{cases}$$

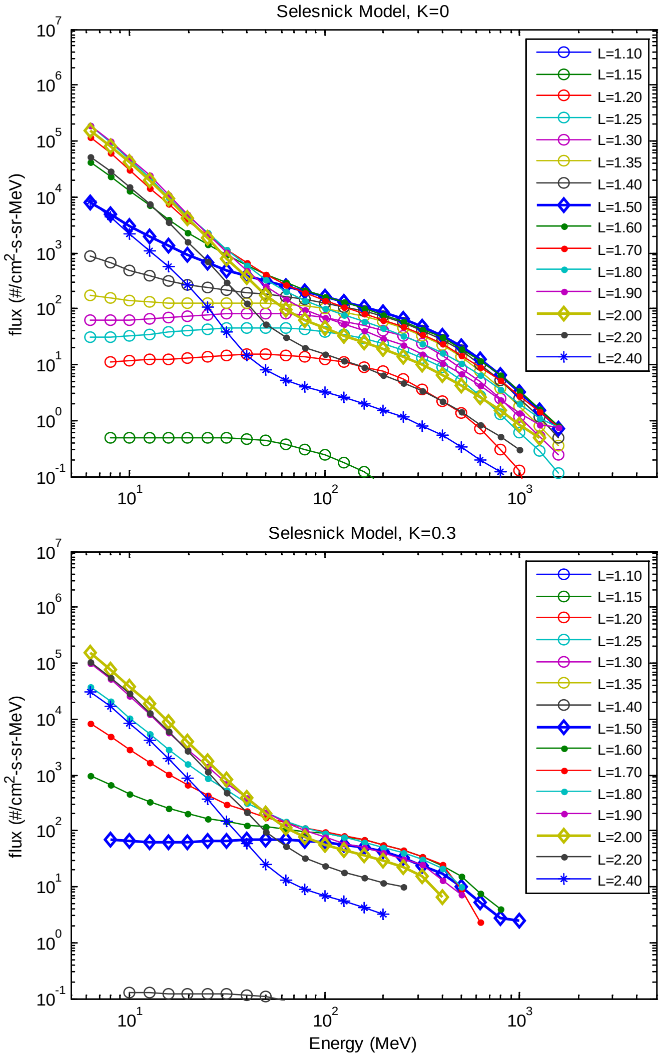
$$E_{break} = 100 \text{ MeV}$$

$$E_0 = 345 \text{ MeV}$$

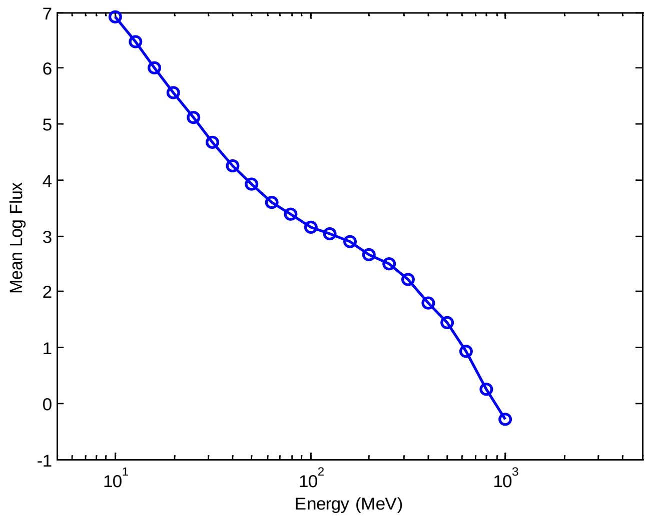
$$j_{break} = \frac{\exp(q_1 - q_2 \ln E_{break})}{\exp\left(-E_{break}/E_0\right)}$$

# Selesnick PCA Model

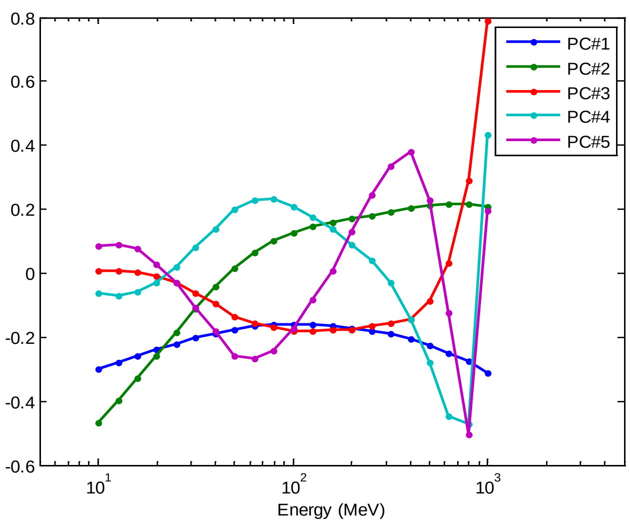
- **Selesnick model has fluxes at fixed values of M, K, L***
- **Fluxes were interpolated to a uniform E grid, then gaps in K and L* were filled in**
- **Although energies extended as low as ~ 1 MeV, 10 MeV was used as a lower limit for PCA**
  - **Below this, not all K/L* values are filled in, resulting in a bias towards higher fluxes**

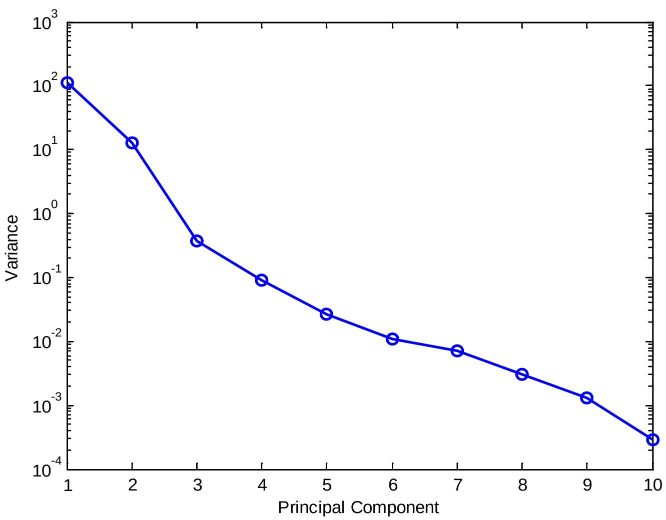


# Principal Components

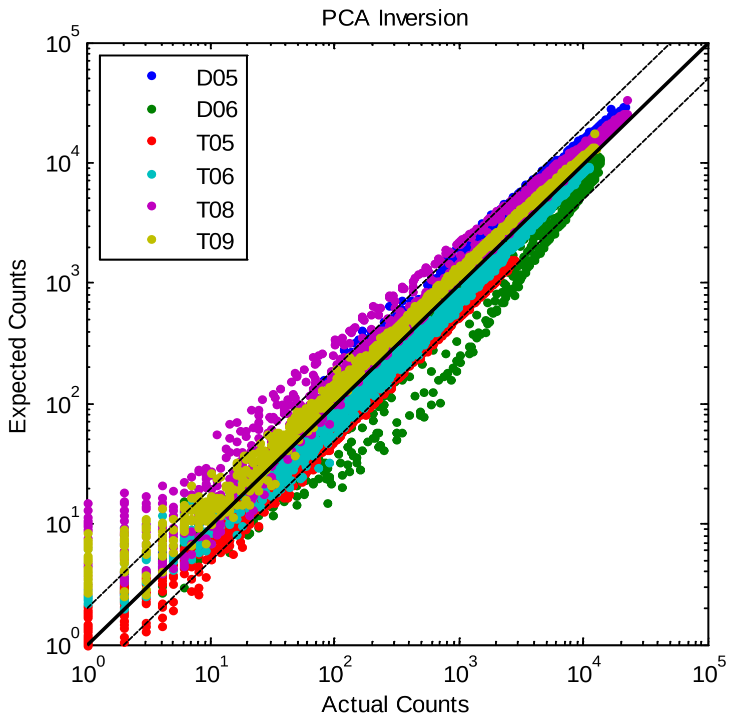


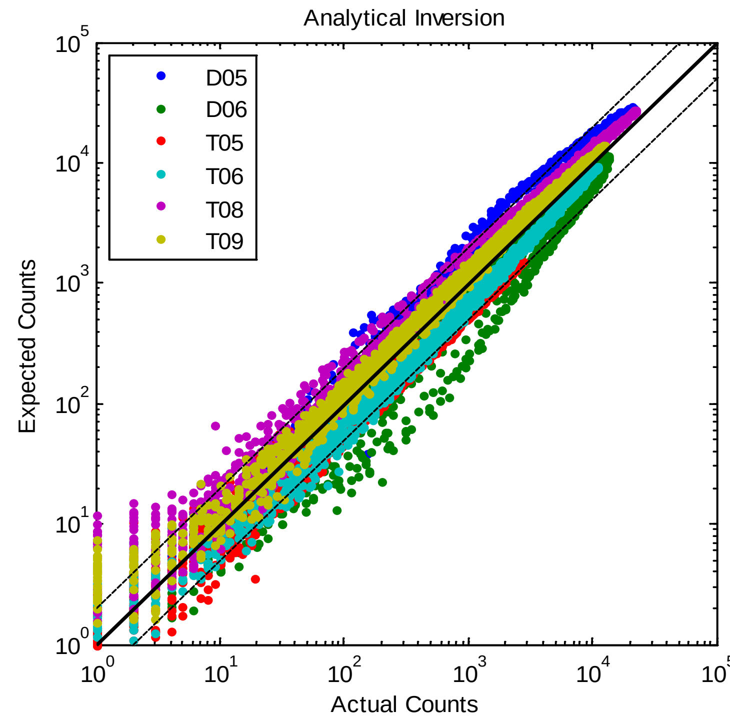

- **PCs well-behaved up through #5 (except near 1000 MeV)**
- **PC#4 and higher contribute very little to variance**



# Inversion Results: Actual Counts vs. Expected Counts

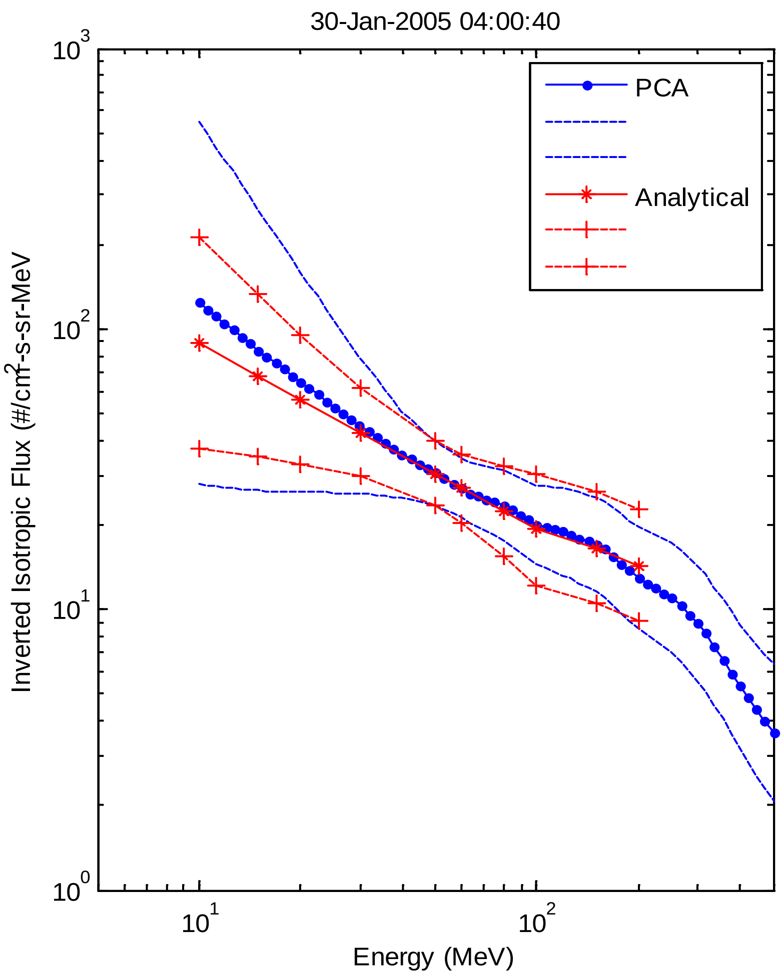



- **Comparisons with analytical inversions used 3 PCs**
- **PCA inversion results in similar reconstruction of expected counts (PCA may be a little better, at least at high count rates)**

# Flux Spectra – Analytical vs. PCA

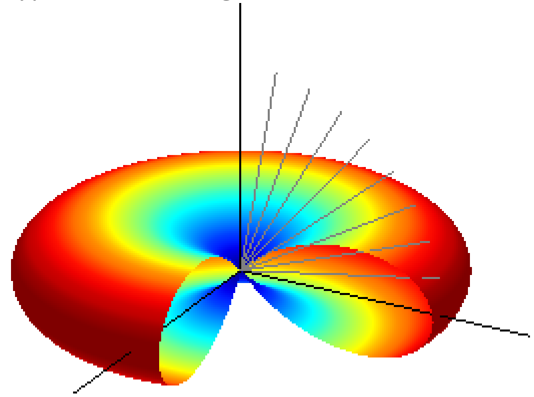
- **"Typical" spectra from analytical and PCA inversions**
- **PCA spectral shape is generally very close to analytical, except near $E_{break}$ (in this example the reverse is true)**
- **Error bars for PCA are not always this bad**

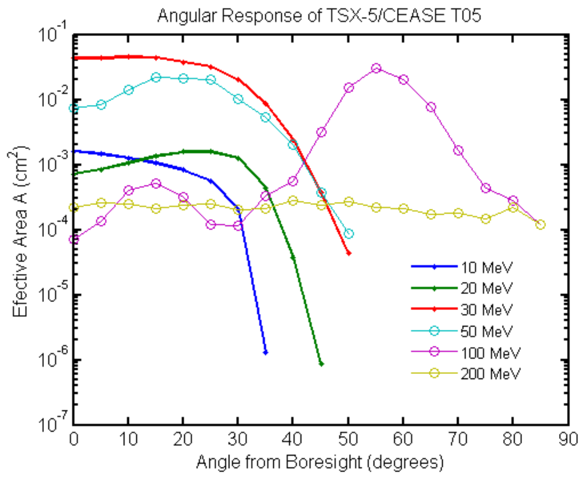


# Angular Correction

- **Much of our data come from wide-angle or omnidirectional detectors, which sample a fraction of the local omnidirectional flux**
- **Need a method to estimate $j_{90}$ from this “semi-omnidirectional” flux**
  - **Particle angular distribution**
  - **Angular response of detector**
- **V1.0 used a correction after performing spectral inversion**
- **For V1.x, we plan to use combined energy/angular inversion as appropriate**

“Typical” Pitch Angle Distribution





# Proton Angular Distribution Function

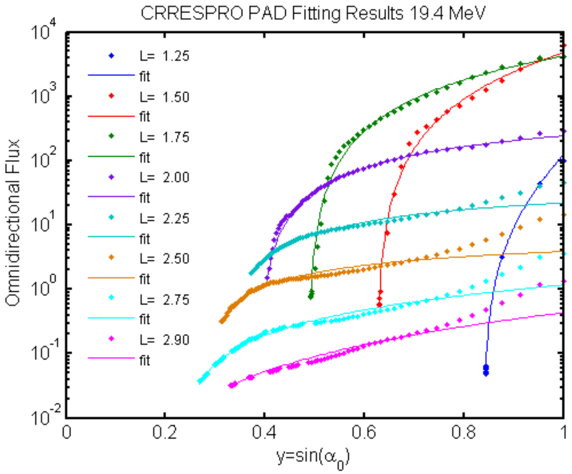
- **Pitch angle distribution based on CRRESPRO model**

$$j = \begin{cases} j_0 \dfrac{(y - y_{LC})^a y^b}{(1 - y_{LC})^a}; & y > y_{LC} \\ 0; & y \le y_{LC} \end{cases}$$

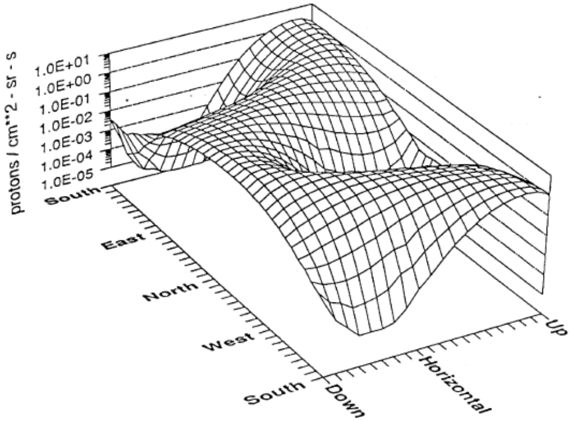
$y = \sin\alpha_0$ (equatorial pitch angle)

$y_{LC} = \sin\alpha_{LC}$ (equatorial loss cone angle)

$a, b, j_0, y_{LC}$ are fitting parameters




- **Azimuthal variation based on Lenchek-Singer, function of**
  - **Atmospheric scale height**
  - **Gyroradius**
  - **Magnetic Inclination**



# Requirements

- **Instrument response functions as function of energy (and angle)**
  - **at least threshold energy & geometric factor**
- **Prior knowledge of spectral shapes**
  - **PCA can provide a rational basis**
  - **For angular inversion, also need PAD**
- **Remember inversion is only valid for range of instrument response**

# Implementation

- **Spectral inversion routines have been implemented in invlib, a C- and MATLAB-callable library (code & documentation available on SourceForge)**
  - **many options for analytical spectral shapes, as well as PCA**
  - **outputs include energy spectra, error bars, expected counts**
- **Used for TSX5/CEASE, HEO/dos, ICO/dos**
  - **protons used Selesnick PCA model**
  - **electrons used PCA model based on CRRES MEA/HEEF**
- **New PCA models have been developed based on AP9 and AE9**

# Questions?